International Journal of Theoretical Physics, Vol. 29, No. 4, 1990

Charged Spherically Symmetric Solution in Mikhail–Wanas Field Theory

Ashish Mazumder[1] **and Dipankar Ray**[2]

Received November 22, 1988; revised December 21, 1988

A set of coupled differential equations obtained by Wanas in the Mikhail–Wanas generalized field theory is completely integrated.

1. INTRODUCTION

Mikhail and Wanas (1977) proposed a generalized field theory using a space admitting absolute parallelism. Wanas (1985) sought the exact solution of the field equations in that theory. The field equations are of the form

$$E_{\mu\nu}=0$$

where $E_{\mu\nu}$ is a second-order nonsymmetric tensor given by

$$E_{\mu\nu}=g_{\mu\nu}L-2L_{\mu\nu}-2g_{\mu\nu}C^{\overset{\sigma}{-}}|_{\sigma}-2C_{\mu}C_{\nu}-2g_{\mu\nu}C^{\varepsilon}\Lambda^{\sigma}_{\varepsilon\nu}+2C_{\underset{+}{\nu}}|_{\mu}-2g^{\sigma\varepsilon}\Lambda_{\underset{+}{\mu}\underset{+}{\nu}\underset{+}{\varepsilon}}|_{\sigma} \tag{1}$$

and

$$L=\Lambda^{\sigma}_{\varepsilon\mu}\Lambda^{\varepsilon}_{\sigma\nu}-C_{\mu}C_{\nu},\qquad L=g^{\mu\nu}L_{\mu\nu},\qquad C_{\mu}=\Lambda^{\varepsilon}_{\mu\varepsilon}$$

$$\Lambda^{\varepsilon}_{\mu\nu}=F^{\varepsilon}_{\mu\nu}-F^{\varepsilon}_{\nu\mu},\qquad \Lambda_{\varepsilon\mu\nu}=g_{\varepsilon\sigma}\Lambda^{\sigma}_{\mu\nu},\qquad F^{\sigma}_{\mu\nu}=\underset{i}{\overset{\sigma}{\lambda}}\underset{i}{\lambda}_{\mu,\nu}$$

Here the vertical bar denotes absolute differentiation using the nonsymmetric connection $F^{\sigma}_{\mu\nu}$. The (+) and the (−) signs are used to distinguish between the two types of absolute derivatives.

[1]166, Dr. A. K. Pal Road, Behala, Calcutta-700034, West Bengal, India.
[2]Department of Mathematics, Jadavpur University, Calcutta-700032, West Bengal, India.

If the structure of space is of the form

$$\underset{0}{\overset{0}{\lambda}} = A, \qquad \underset{0}{\overset{\alpha}{\lambda}} = D, \qquad \underset{a}{\overset{\alpha}{\lambda}} = \underset{a}{\overset{\alpha}{\delta}} B \tag{2}$$

where A, B, and D are functions of r (i.e., $\chi^\alpha\chi^\alpha$), then Wanas (1985) showed that the field equations reduce to

$$\frac{B^2+D^2r^2}{A^2}\left[b(r)+\frac{D^2r^2}{B^2}\,l(r)\right]=0 \tag{3a}$$

$$\frac{Dr}{A}\left[b(r)+\frac{D^2r^2}{B^2}\,l(r)\right]=0 \tag{3b}$$

$$-\frac{B'^2}{B^2}+\frac{2}{r}\left(\frac{B'}{B}+\frac{A'}{A}\right)-\frac{2A'B'}{AB}-l(r)\,\frac{D^2r^2}{B^2}=0 \tag{3c}$$

$$\begin{aligned}\frac{D^2r^2}{B^2}\Bigg[&-l(r)+\frac{A''}{A}-\frac{D''}{D}+\frac{4}{r}\left(\frac{A'}{A}-\frac{D'}{D}\right)-\frac{2A'^2}{A^2}-\frac{D'^2}{D^2}\\ &+3\left(\frac{A'D'}{AD}-\frac{A'B'}{AB}+\frac{B'D'}{BD}\right)\Bigg]+\frac{A''}{A}+\frac{B''}{B}-\frac{2A'^2}{A^2}-\frac{B'^2}{B^2}\\ &+\frac{1}{r}\left(\frac{A'}{A}+\frac{B'}{B}\right)=0\end{aligned} \tag{3d}$$

where

$$\begin{aligned}b(r)&=3\frac{B'^2}{B^2}-\frac{4}{r}\frac{B'}{B}-\frac{2B''}{B}\\ l(r)&=5\frac{B'^2}{B^2}-\frac{8}{r}\frac{B'}{B}-\frac{2B''}{B}+\frac{2}{r}\frac{D'}{D}-\frac{2B'D'}{BD}+\frac{3}{r^2}\end{aligned} \tag{3e}$$

where the prime denotes differentiation with respect to r.

Wanas (1985) solved the set of equations (3) under the assumption

$$\left(\frac{A'}{A}+\frac{2B'}{B}\right)\left(\frac{B'D^2}{B}-DD'-\frac{3D^2}{r}\right)=0 \tag{4}$$

The purpose of the present note is to show that the set of equations (3) can be reduced to two equations, one expressing A in terms of r, B, and dB/dr, and the other expressing D in terms of A, B, and r, which means that equations (3) are solved for arbitrary choice of $B(r)$.

2. REDUCTION OF EQUATIONS (3)

Eliminating $l(r)$ from (3b) and (3c) and substituting $b(r)$ from (3e), we get

$$\frac{B'^2}{B^2}-\frac{1}{r^2}\frac{B'}{B}-\frac{B''}{B}+\frac{1}{r}\frac{A'}{A}-\frac{A'B'}{AB}=0 \tag{5}$$

Equation (5) can be readily integrated. Integrating equation (5) and putting that into equations (3), one can reduce equations (3) to the following three equations:

$$A=\frac{K_1}{1-\alpha_z} \tag{6a}$$

$$\left[1+\frac{1}{\beta}\exp(2\alpha-2z)\right](\alpha_z^2-2\alpha_z-2\alpha_{zz})+(1-\alpha_z)\times\left(\frac{\beta_z}{\beta}-2\alpha_z+2\right)+1=0 \tag{6b}$$

and

$$\left[1+\frac{1}{\beta}\exp(2\alpha-2z)\right]\left(\frac{\alpha_{zzz}}{1-\alpha_z}+\alpha_{zz}\right)+\left(\frac{3\alpha_{zz}}{1-\alpha_z}+\alpha_z-1\right)\times\left(\frac{\beta_z}{2\beta}-\alpha_z+1\right)+\alpha_{zz}-\frac{1}{2}\left(\frac{\beta_z}{\beta}\right)_z-2\left(\frac{\beta_z}{2\beta}-\alpha_z+1\right)^2=0 \tag{6c}$$

where

$$B=\exp(\alpha) \tag{6d}$$

$$D^2=\beta \tag{6e}$$

$$z=\ln r \tag{6f}$$

and K_1 is a constant.

However, differentiation of equation (6b) with respect to z immediately gives equation (6c). Hence, one is basically left with equations (6a) and (6b). Now equation (6b) can be integrated to give

$$\beta(1-\alpha_z)^2\exp[3(z-\alpha)]+\alpha_z(\alpha_z-2)\exp(z-\alpha)=K_2$$

where K_2 is a constant, i.e.,

$$D^2=\frac{1}{(1-\alpha_z)^2\exp 3(z-\alpha)}[K_2-\alpha_z(\alpha_z-2)\exp(z-\alpha)] \tag{6b'}$$

Thus, equations (3) have been reduced to two equations, namely (6a) and (6b′). In terms of the original variables these equations take the following form:

$$A = \frac{K_1}{1 - rB'/B} \tag{7a}$$

$$D^2 = \frac{B''}{K_1^2 r^3}(K_2 B + r)A + K_1^2 r \tag{7b}$$

where K_1 and K_2 are constants.

3. CONCLUSIONS

In summary, the field equations in the Mikhail and Wanas (1977) generalized field theory for the structure of space of the form in equation (2), namely equations (3), have been reduced to equations (7a) and (7b), which give A and D for arbitrary $B(r)$. Once A, B, and D are known, the structure of space is given through equations (2) and (1).

Solutions for equations (3) obtained by Wanas (1985) under the assumption of equation (4) can now be trivially verified for equations (7).

REFERENCES

Mikhail, F. I., and Wanas, M. I. (1977). *Proceedings of the Royal Society of London, Series A*, **356**, 471.

Wanas, M. I. (1985). *International Journal of Theoretical Physics*, **24**, 642.